

# On k-Heronian Mean Labeling

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**ABSTRACT:** In this paper, we introduce k-Heronian mean labeling and investigate k-Heronian mean labeling of Path,  $P_n \odot mK_1$ ;  $1 \leq m \leq 4$  and Ladder.

**KEYWORDS:** k-Heronian mean labeling, k-Heronian mean graph, Path,  $P_n \odot mK_1$ ;  $1 \leq m \leq 4$ , Ladder.

## I. INTRODUCTION

In this paper, all graphs are finite, undirected and simple graphs. Terms not defined here are used in the sense of Harary [2]. Graph labeling is first introduced by Rosa in 1967. The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [5]. The concept of Heronian mean labeling was introduced and studied by S. S. Sandhaya and others [3] [4]. Here we investigate k – Heronian mean labeling of Path,  $P_n \odot nK_1$ ;  $1 \leq n \leq 4$ , and Ladder. For brevity, we use k-HML for k-Heronian mean labeling and k is any positive integer.

## II. MAIN RESULTS

### Definition 2.1:

A graph  $G=(V,E)$  with p vertices and q edges is said to be a **k-Heronian Mean graph** if it is possible to label the vertices  $x \in V$  with distinct labels  $f(x)$  from  $k, k+1, k+2, \dots, k+q$  in such a way that when each edge  $e=uv$  is labeled with,  $f^*(e) = \left\lfloor \frac{f(u)+f(v)+\sqrt{f(u)f(v)}}{3} \right\rfloor$  or  $\left\lceil \frac{f(u)+f(v)+\sqrt{f(u)f(v)}}{3} \right\rceil$ , then the resulting edge labels are distinct. In this case  $f$  is called a **k-Heronian Mean labeling** of G.

### Theorem 2.2:

Any path  $P_n$  is a k-Heronian mean graph, for all  $n \geq 2$ .

#### Proof:

Let  $V(P_n) = \{u_i; 1 \leq i \leq n\}$  and  $E(P_n) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\}$  be the vertices and edges of  $P_n$  respectively.

Define  $f: V(P_n) \rightarrow \{k, k + 1, k + 2, \dots, k + n - 1\}$  by

$$f(u_i) = k + i - 1; 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e_i) = k + i - 1; 1 \leq i \leq n - 1.$$

Here  $p = n$  and  $q = n-1$ .

Clearly,  $f$  is k-Heronian mean labeling of  $P_n$ .

Hence  $P_n$  is a k-Heronian mean graph, for all  $n \geq 2$ .

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**Example 2.3:**

50-Heronian mean labeling of  $P_5$  is given in the figure 2.1:

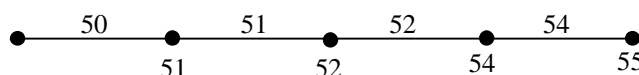


Figure 2.1: 50-HML of  $P_5$

**Theorem 2.3:**

Any comb,  $P_n \odot K_1$  is a k-Heronian mean graph, for all  $n \geq 2$ .

**Proof:**

Let  $V(P_n \odot K_1) = \{u_i, v_i; 1 \leq i \leq n\}$  and  $E(P_n \odot K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{e'_i = (u_i, v_i); 1 \leq i \leq n\}$  be the vertices and edges of  $P_n \odot K_1$  respectively.

Define  $f: V(P_n \odot K_1) \rightarrow \{k, k + 1, k + 2, \dots, k + 2n - 1\}$  by

$$f(u_i) = k + 2i - 2; 1 \leq i \leq n,$$

$$f(v_i) = k + 2i - 1; 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e_i) = k + 2i - 2; 1 \leq i \leq n,$$

$$f^*(e'_i) = k + 2i - 1; 1 \leq i \leq n - 1.$$

Here  $p = 2n$  and  $q = 2n - 1$ .

Clearly,  $f$  is k-Heronian mean labeling of  $P_n \odot K_1$ .

Hence  $P_n \odot K_1$  is a k-Heronian mean graph, for all  $n \geq 2$ .

**Example 2.4:**

75-Heronian mean labeling of  $P_5 \odot K_1$  is given in the figure 2.2:

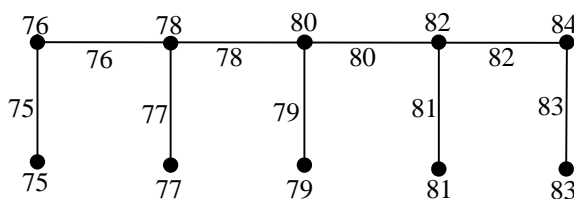


Figure 2.2: 75-HML of  $P_5 \odot K_1$

**Theorem 2.5**

The graph  $P_n \odot 2K_1$  is a k-Heronian mean graph, for all  $n \geq 2$ .

**Proof:**

Let  $V(P_n \odot 2K_1) = \{u_i, v_i, w_i; 1 \leq i \leq n\}$  and  $E(P_n \odot 2K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{e'_i = (u_i, v_i); 1 \leq i \leq n\} \cup \{e''_i = (u_i, w_i); 1 \leq i \leq n\}$  be the vertices and edges of  $P_n \odot 2K_1$  respectively.

Define  $f: V(P_n \odot 2K_1) \rightarrow \{k, k + 1, k + 2, \dots, k + 3n - 1\}$  by

$$f(u_i) = k + 3i - 2; 1 \leq i \leq n,$$

$$f(v_i) = k + 3i - 3; 1 \leq i \leq n,$$

$$f(w_i) = k + 3i - 1; 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e_i) = k + 3i - 1; 1 \leq i \leq n - 1,$$

$$f^*(e'_i) = k + 3i - 3; 1 \leq i \leq n,$$

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$$f^*(e_i'') = k + 3i - 2; \quad 1 \leq i \leq n.$$

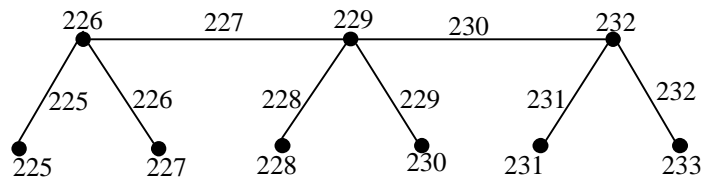
Here  $p = 3n$  and  $q = 3n-1$ .

Clearly,  $f$  is  $k$ -Heronian mean labeling of  $P_n \odot 2K_1$ .

Hence  $P_n \odot 2K_1$  is a  $k$ -Heronian mean graph, for all  $n \geq 2$ .

### Example 2.6:

225-Heronian mean labeling of  $P_3 \odot 2K_1$  is given in the figure 2.3:



**Figure 2.3: 225-HML of  $P_3 \odot 2K_1$**

### Theorem 2.7:

The graph  $P_n \odot 3K_1$  is a  $k$ -Heronian mean graph, for all  $n \geq 2$ .

#### Proof:

Let  $V(P_n \odot 3K_1) = \{u_i, v_i, w_i, x_i; 1 \leq i \leq n\}$  and  $E(P_n \odot 3K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup \{e_i' = (u_i, v_i); 1 \leq i \leq n\} \cup \{e_i'' = (u_i, w_i); 1 \leq i \leq n\} \cup \{e_i''' = (u_i, x_i); 1 \leq i \leq n\}$  be the vertices and edges of  $P_n \odot 3K_1$  respectively.

Define  $f: V(P_n \odot 3K_1) \rightarrow \{k, k+1, k+2, \dots, k+4n-1\}$  by

$$f(u_i) = k + 4i - 3; \quad 1 \leq i \leq n,$$

$$f(v_i) = k + 4i - 4; \quad 1 \leq i \leq n,$$

$$f(w_i) = k + 4i - 3; \quad 1 \leq i \leq n,$$

$$f(x_i) = k + 4i - 1; \quad 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e_i) = k + 4i - 1; \quad 1 \leq i \leq n-1,$$

$$f^*(e_i') = k + 4i - 4; \quad 1 \leq i \leq n,$$

$$f^*(e_i'') = k + 4i - 3; \quad 1 \leq i \leq n,$$

$$f^*(e_i''') = k + 4i - 2; \quad 1 \leq i \leq n.$$

Here  $p = 4n$  and  $q = 4n-1$ .

Clearly,  $f$  is  $k$ -Heronian mean labeling of  $P_n \odot 3K_1$ .

Hence  $P_n \odot 3K_1$  is a  $k$ -Heronian mean graph, for all  $n \geq 2$ .

### Example 2.8:

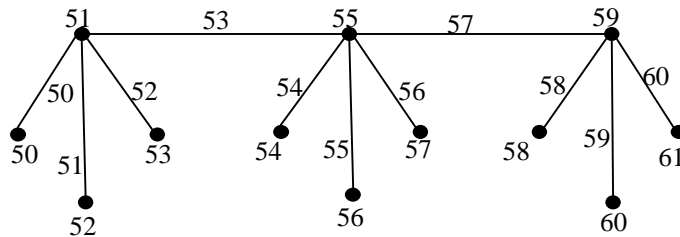
50-Heronian mean labeling of  $P_3 \odot 3K_1$  is given in the figure 2.4:

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**Figure 2.3: 50-HML of  $P_3 \odot 3K_1$**

**Theorem 2.9:**

The graph  $P_n \odot 4K_1$  is a k-Heronian mean graph, for all  $n \geq 2$ .

**Proof:**

Let  $V(P_n \odot 4K_1) = \{u_i, v_i, w_i; 1 \leq i \leq n\}$  and  $E(P_n \odot 4K_1) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{e'_i = (u_i, v_i); 1 \leq i \leq n\} \cup \{e''_i = (u_i, w_i); 1 \leq i \leq n\} \cup \{e'''_i = (u_i, x_i); 1 \leq i \leq n\} \cup \{e^{iv}_i = (u_i, y_i); 1 \leq i \leq n\}$  be the vertices and edges of  $P_n \odot 4K_1$  respectively.

Define  $f: V(P_n \odot 4K_1) \rightarrow \{k, k + 1, k + 2, \dots, k + 5n - 1\}$  by

$$f(u_i) = k + 5i - 4; \quad 1 \leq i \leq n,$$

$$f(v_i) = k + 5i - 5; \quad 1 \leq i \leq n,$$

$$f(w_i) = k + 5i - 3; \quad 1 \leq i \leq n,$$

$$f(x_i) = k + 5i - 2; \quad 1 \leq i \leq n,$$

$$f(y_i) = k + 5i - 1; \quad 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e_i) = k + 5i - 1; \quad 1 \leq i \leq n - 1,$$

$$f^*(e'_i) = k + 5i - 5; \quad 1 \leq i \leq n,$$

$$f^*(e''_i) = k + 5i - 4; \quad 1 \leq i \leq n,$$

$$f^*(e'''_i) = k + 5i - 3; \quad 1 \leq i \leq n,$$

$$f^*(e^{iv}_i) = k + 5i - 2; \quad 1 \leq i \leq n.$$

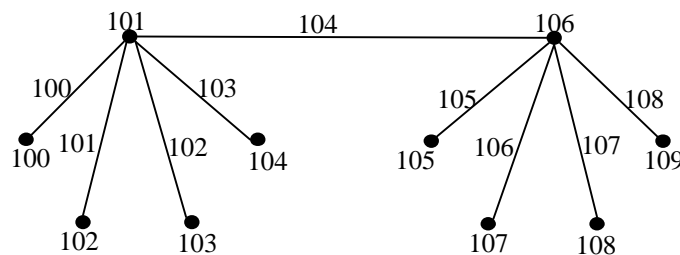
Here  $p = 5n$  and  $q = 5n - 1$ .

Clearly,  $f$  is k-Heronian mean labeling of  $P_n \odot 4K_1$ .

Hence  $P_n \odot 4K_1$  is a k-Heronian mean graph, for all  $n \geq 2$ .

**Example 2.10:**

50-Heronian mean labeling of  $P_3 \odot 4K_1$  is given in the figure 2.5:



**Figure 2.5: 100-HML of  $P_2 \odot 4K_1$**

**Theorem 2.5**

Any Ladder,  $L_n$  is a k-Heronian mean graph, for all  $n \geq 2$ .

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**Proof:**

Let  $V(L_n) = \{u_i, v_i; 1 \leq i \leq n\}$  and  $E(L_n) = \{e_i = (u_i, v_i); 1 \leq i \leq n\} \cup \{e'_i = (u_i, u_{i+1}); 1 \leq i \leq n - 1\} \cup \{e''_i = (v_i, v_{i+1}); 1 \leq i \leq n - 1\}$  be the vertices and edges of  $L_n$  respectively.

Define  $f: V(L_n) \rightarrow \{k, k + 1, k + 2, \dots, k + 3n - 2\}$  by

$$f(u_i) = k + 3i - 3; \quad 1 \leq i \leq n,$$

$$f(v_i) = k + 3i - 2; \quad 1 \leq i \leq n.$$

Now the induced edge labels are

$$f^*(e_i) = k + 3i - 3; \quad 1 \leq i \leq n,$$

$$f^*(e'_i) = k + 3i - 2; \quad 1 \leq i \leq n - 1,$$

$$f^*(e''_i) = k + 3i - 1; \quad 1 \leq i \leq n - 1.$$

Here  $p = 2n$  and  $q = 3n - 2$ .

Clearly,  $f$  is  $k$ -Heronian mean labeling of  $L_n$ .

Hence  $L_n$  is a  $k$ -Heronian mean graph, for all  $n \geq 2$ .

**Example 2.12:**

200-Heronian mean labeling of  $L_5$  is given in the figure 2.6:

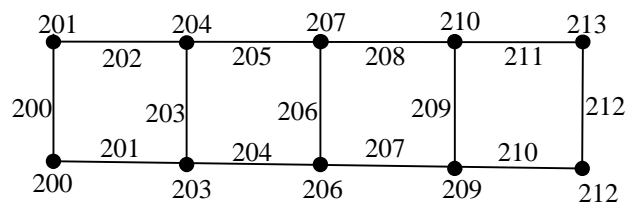


Figure 2.6: 200-HML of  $L_5$

### III. CONCLUSION

Graph labeling has its own applications in communication networks and astronomy, so enormous types of labeling have grown. Towards this  $k$ -Heronian mean labeling is also a kind of labeling which an extension of Heronian mean labeling. In this paper we discussed  $k$ -Heronian mean labeling of the graphs, Path,  $P_n \odot mK_1$ ;  $1 \leq m \leq 4$  and Ladder.

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